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Determining the Value of a Prototype

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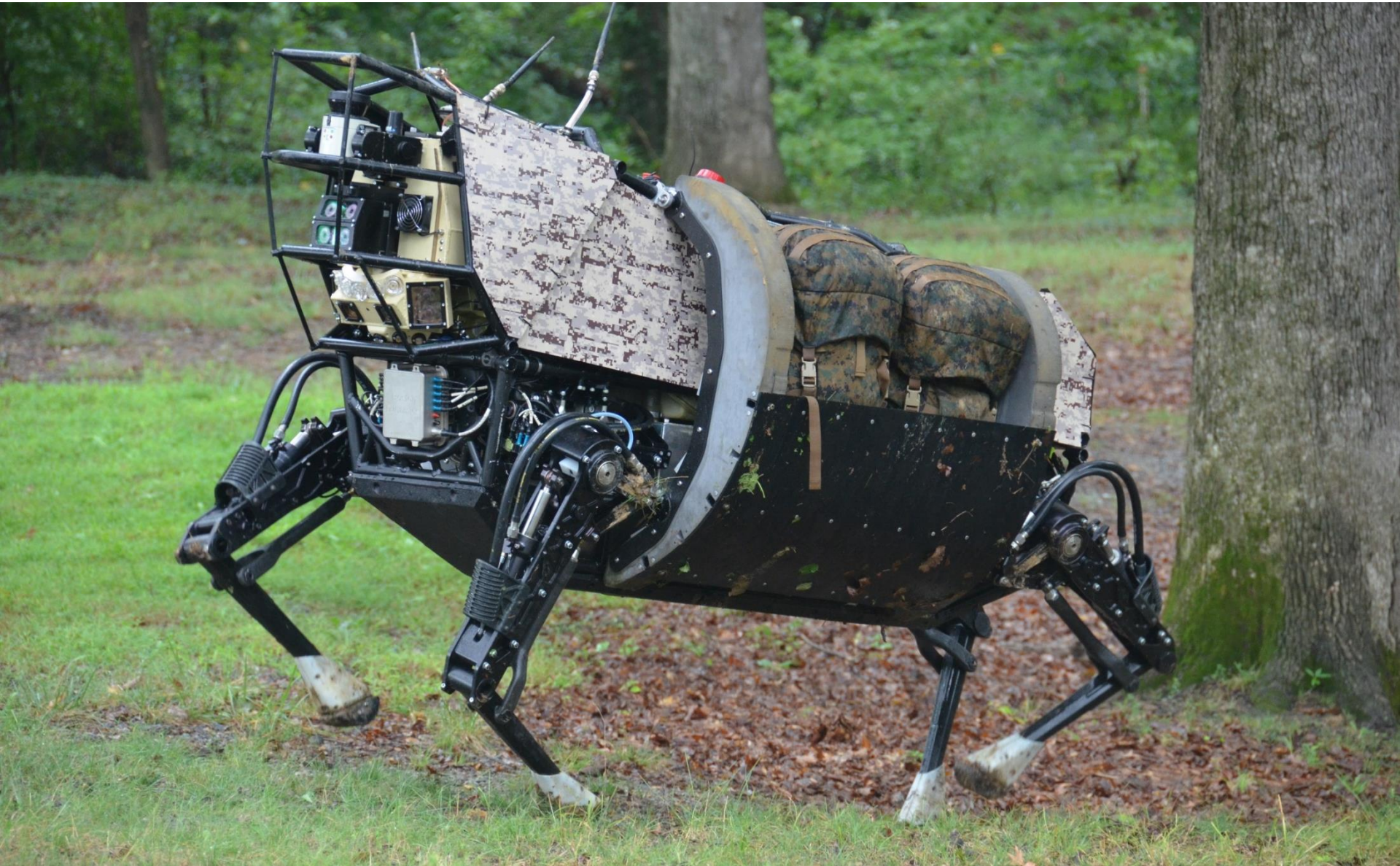


Determining the Value of a Prototype

Zachary Strider McGregor-Dorsey

April 26, 2017

What are prototypes worth?



Setting the Stage – A program is proposed

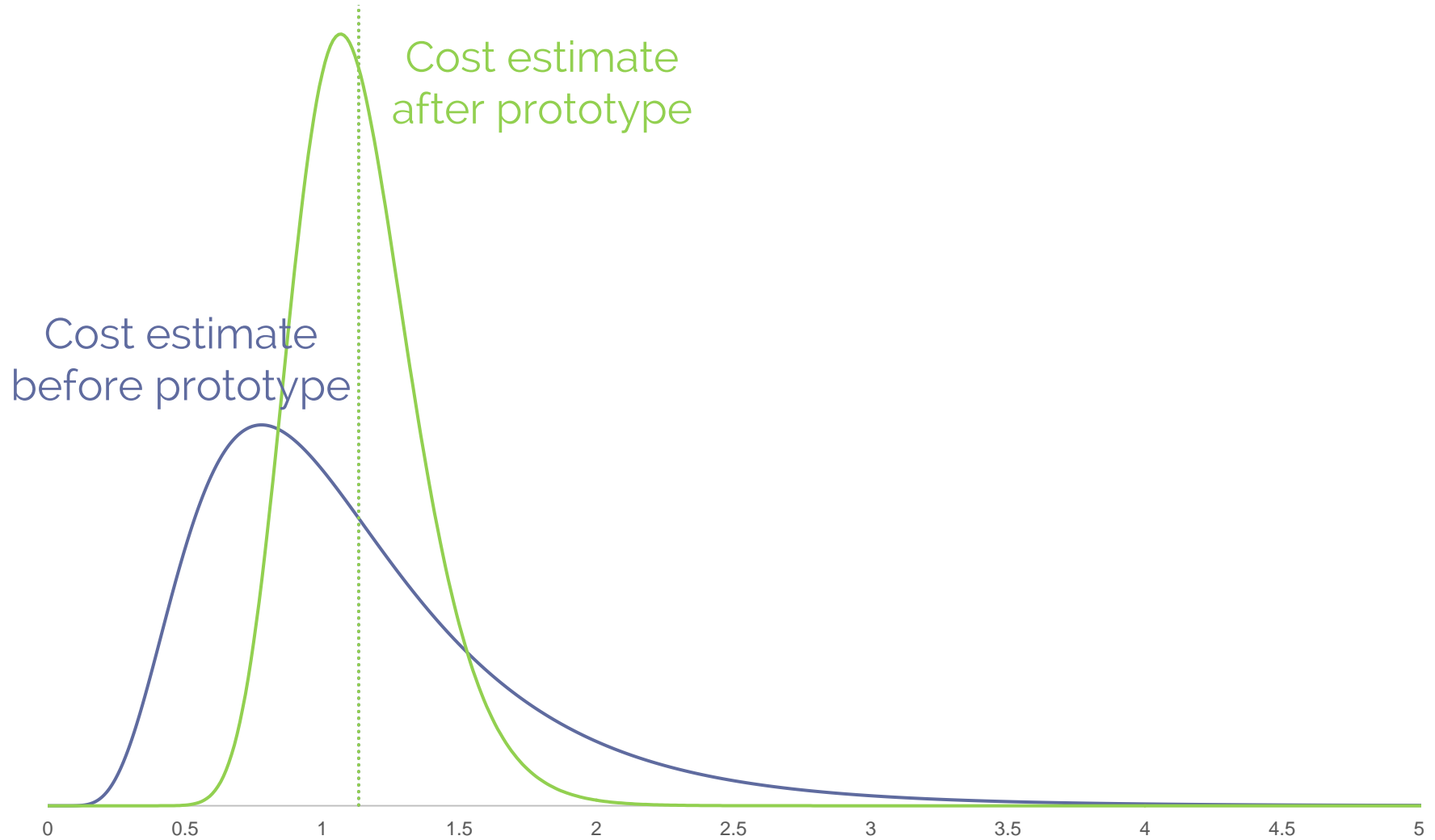
A decisionmaker has three options:

1. Start the program
2. Do not start the program
3. Make a prototype, and then start or do not start the program

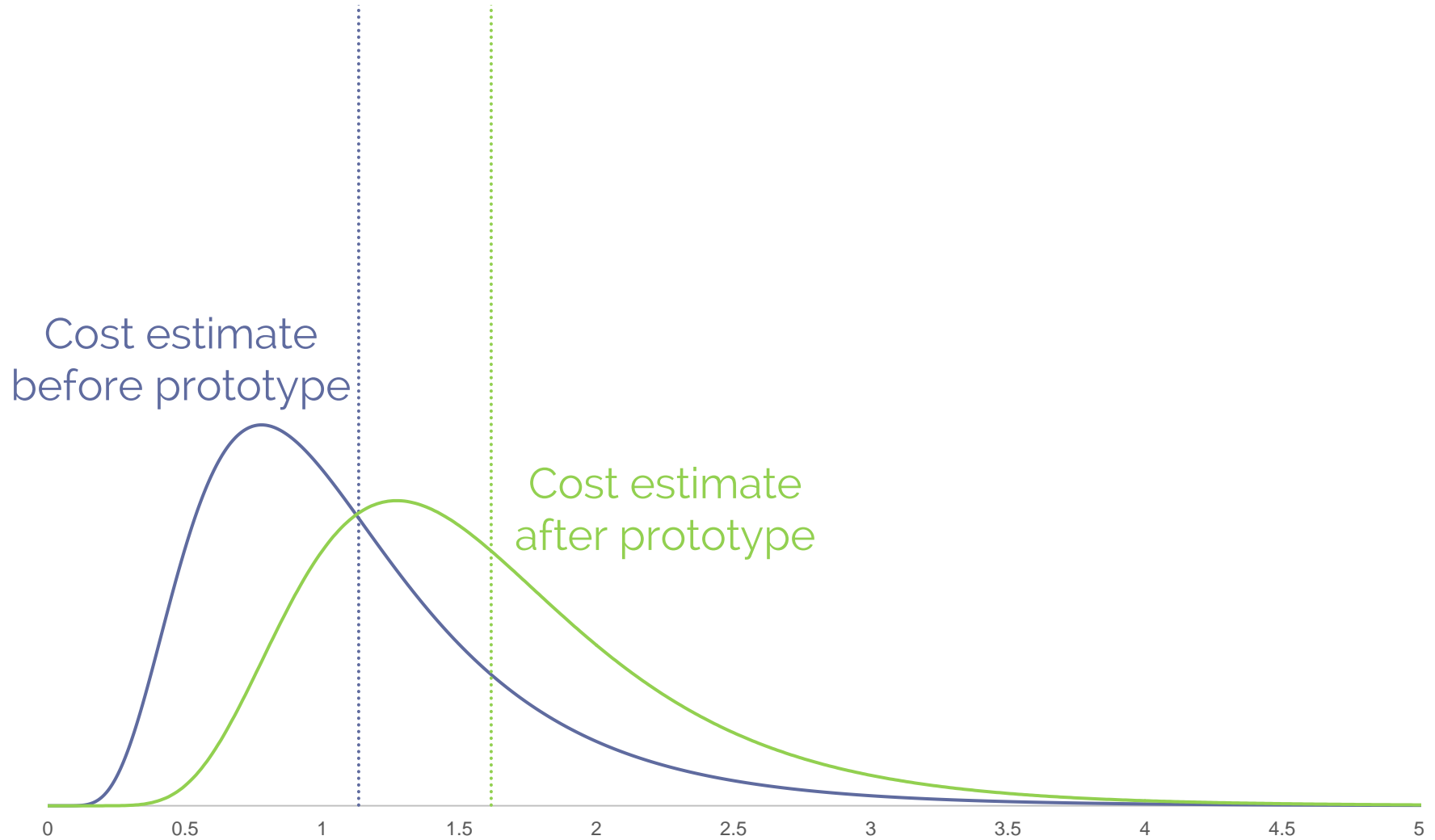
An alternative program option exists with a known cost

Cost is the discriminator

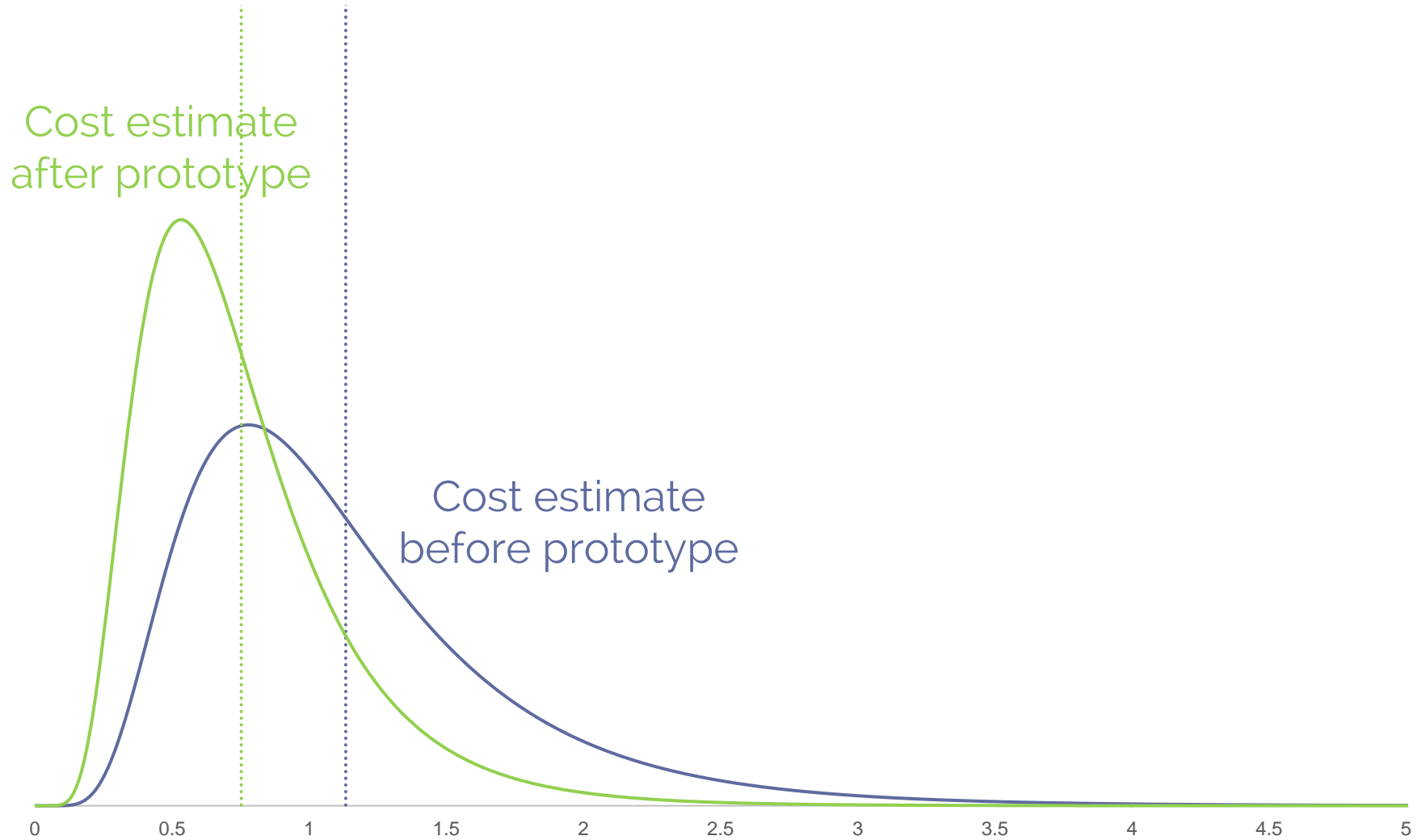
Prototypes give us information



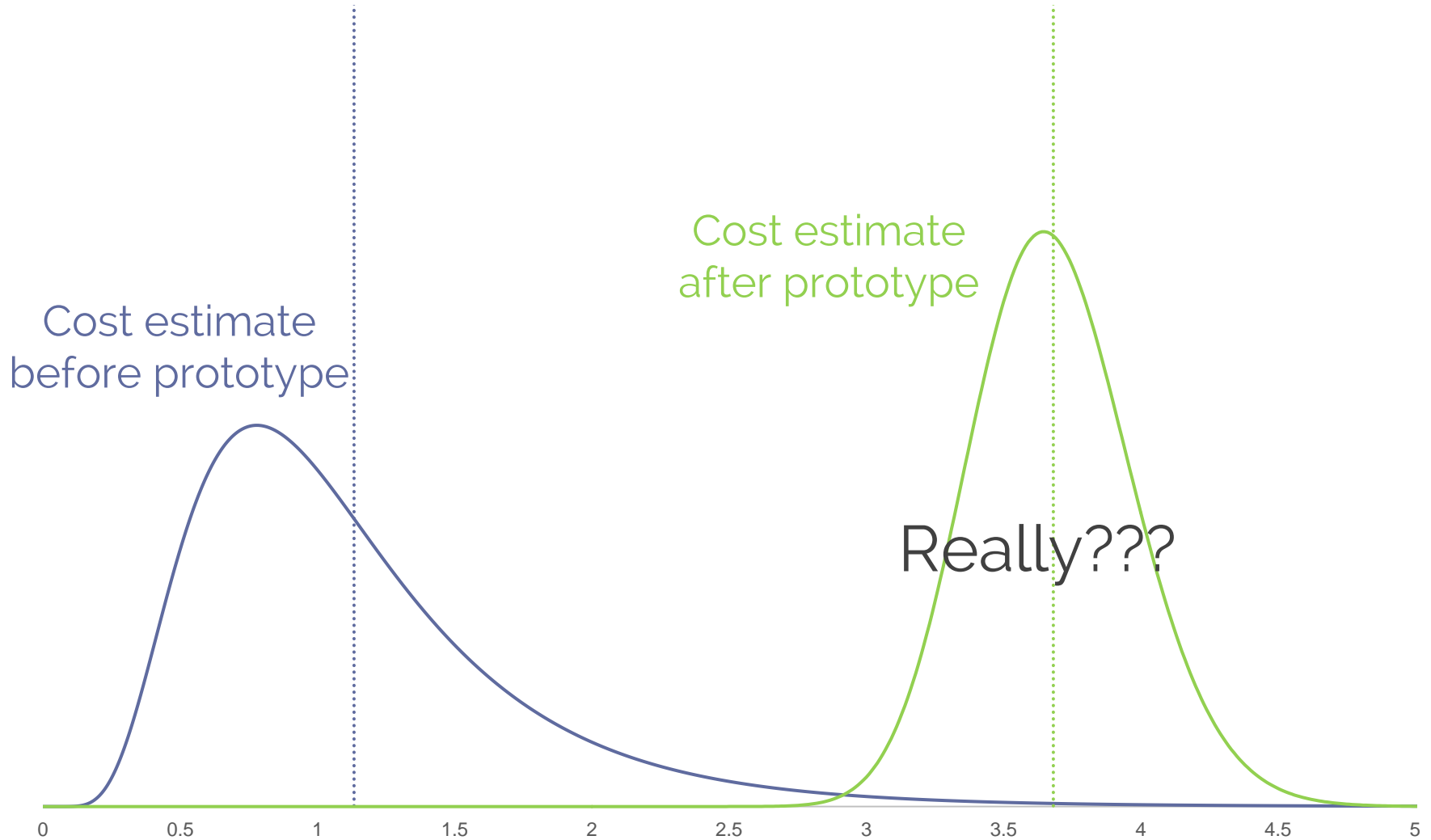
Prototypes give us information



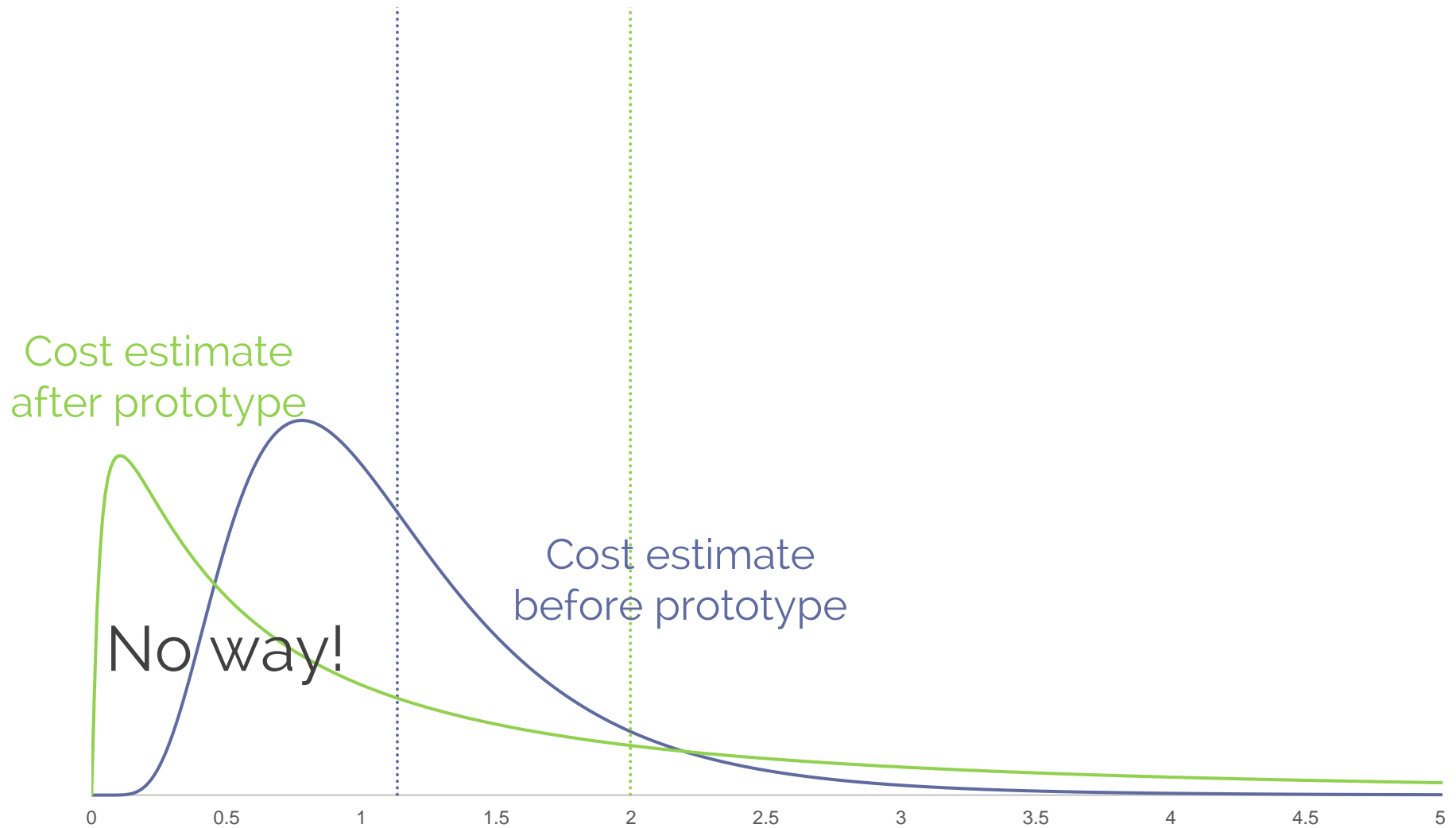
Prototypes give us information



Prototypes give us information...but we know a little of what to expect



Prototypes give us information...but we know a little of what to expect



The Preposterior Distribution

The distribution of the means of all possible cost distributions after a prototype outcome

Properties of the Preposterior Distribution

The mean (expected value):

$$E[\textit{preposterior}] = E[\textit{prior}]$$

The mean of the preposterous is equal to
the mean of the prior

Properties of the Preposterior Distribution

The mean (expected value):

$$E[\textit{preposterior}] = E[\textit{prior}]$$

The variance:

$$\text{Var}[\textit{preposterior}] = \text{Var}[\textit{prior}] - E_{d \in \textit{posteriors}}[\text{Var}[d]]$$

The variance of the preposterous is equal to
the variance of the prior less
the mean of the posterior variances

Properties of the Preposterior Distribution

The mean (expected value):

$$E[\textit{preposterior}] = E[\textit{prior}]$$

The variance:

$$\text{Var}[\textit{preposterior}] = \text{Var}[\textit{prior}] - E_{d \in \textit{posteriors}}[\text{Var}[d]]$$

The shape:

$$\text{Dist}[\textit{preposterior}] \rightarrow \text{Dist}[\textit{prior}] \text{ as } \textit{experiments} \rightarrow \infty$$

The shape of the preposterous approaches that of the prior with increased experimental data

Properties of the Preposterior Distribution

The mean (expected value):

$$E[\textit{preposterior}] = E[\textit{prior}]$$

The variance:

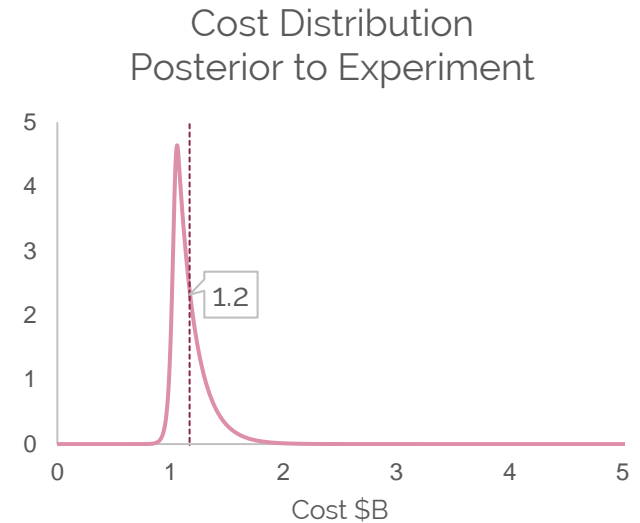
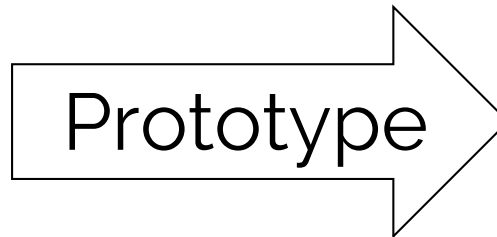
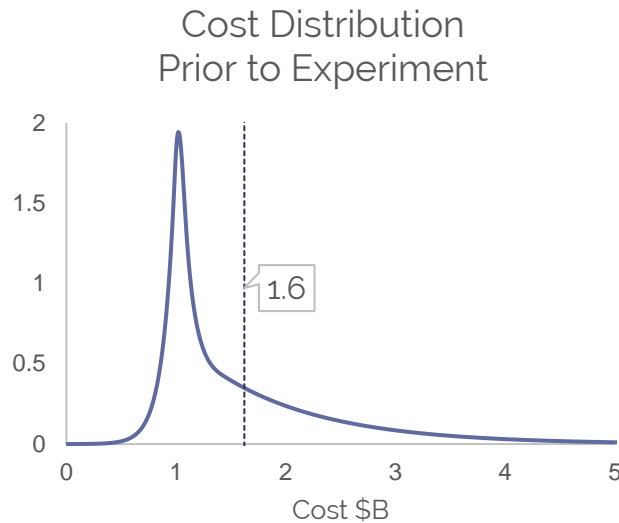
$$\text{Var}[\textit{preposterior}] = \text{Var}[\textit{prior}] - E_{d \in \textit{posteriors}}[\text{Var}[d]]$$

The shape:

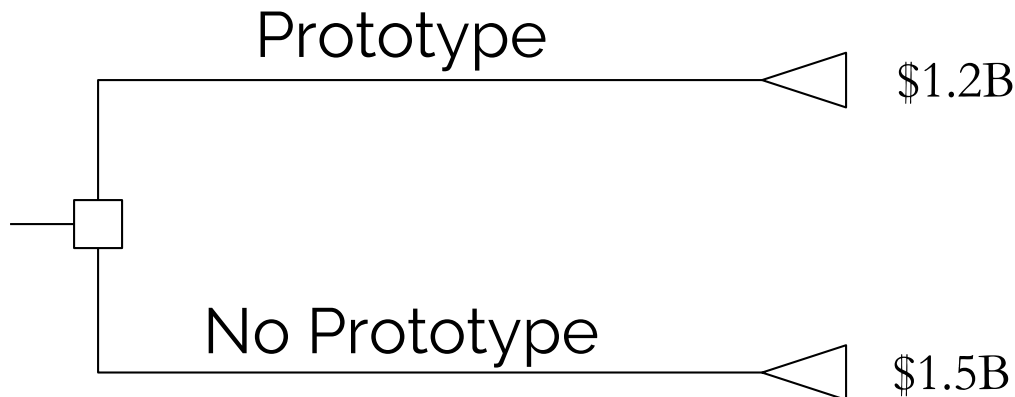
$$\text{Dist}[\textit{preposterior}] \rightarrow \text{Dist}[\textit{prior}] \text{ as } \textit{experiments} \rightarrow \infty$$

In other words, the preposterior distribution
looks a lot like the prior distribution

How to use the preposterior distribution



Suppose the alternative costs \$1.5B

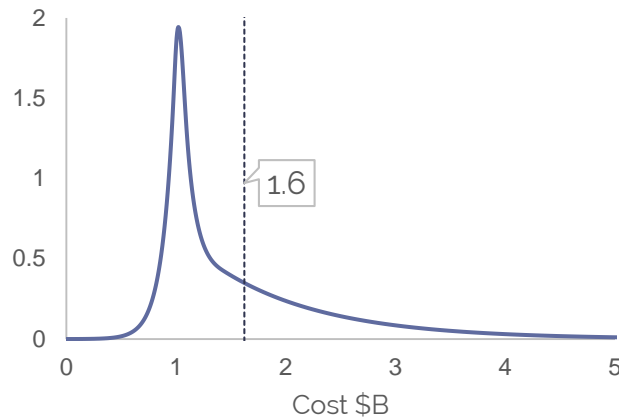


$$\Delta = \$0.3B$$

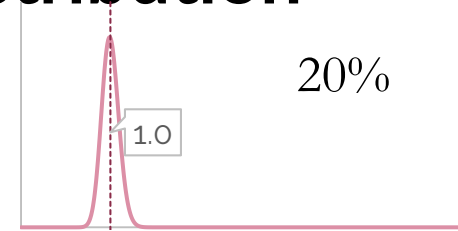
Value of information
from prototype

How to use the preposterior distribution

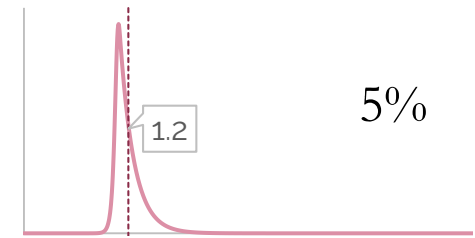
Cost Distribution
Prior to Experiment



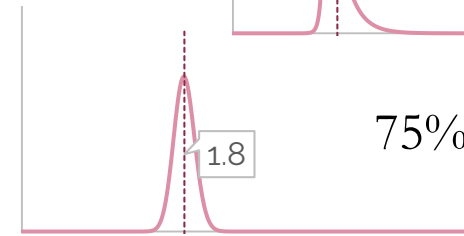
Prototype



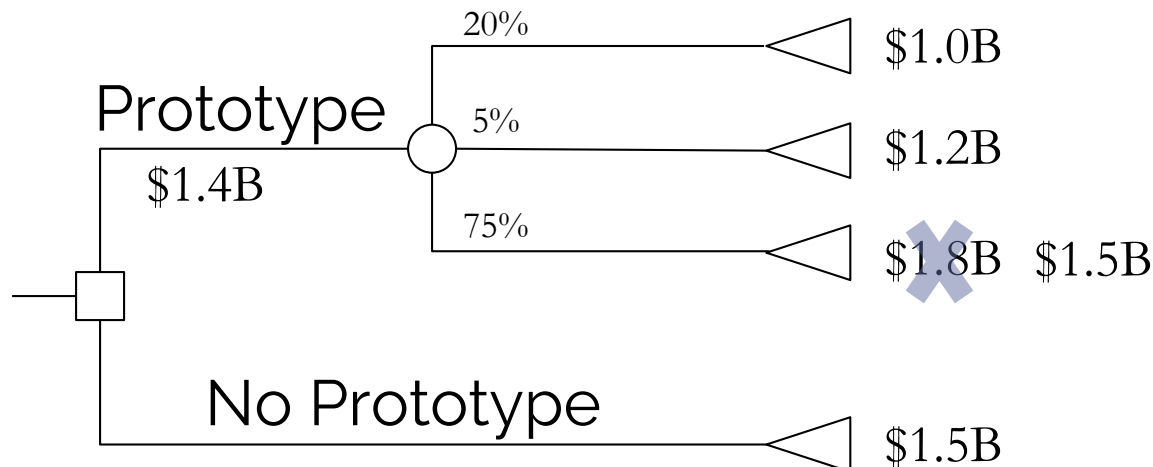
20%



5%



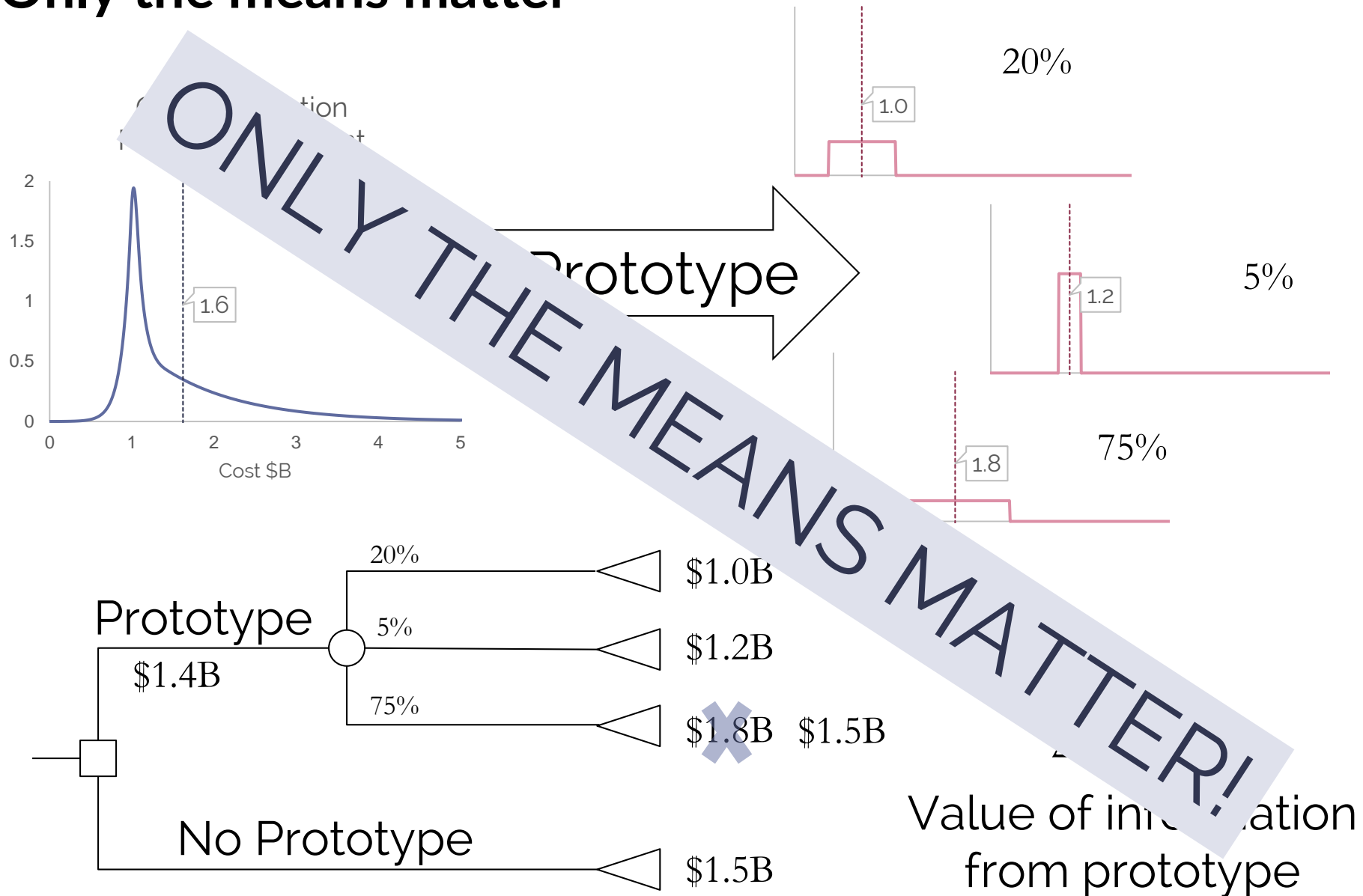
75%



$$\Delta = \$0.1B$$

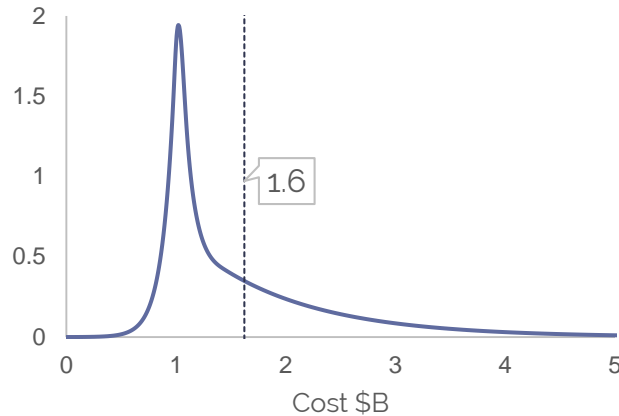
Value of information
from prototype

Only the means matter

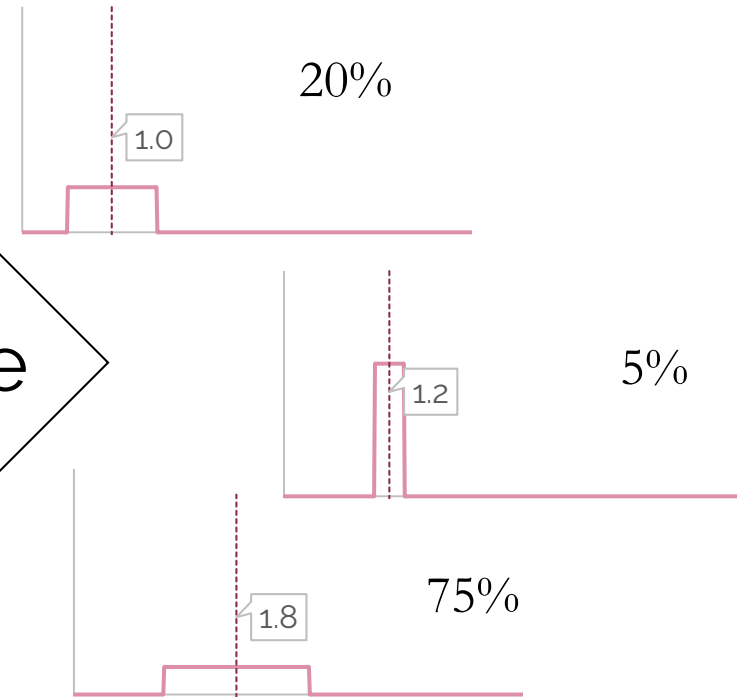


Prepostrior to the rescue

Cost Distribution
Prior to Experiment



Prototype



Prior distribution

Posterior distributions

We know the mean of these means!

We can even make a decent
guess as to the shape of the
distribution of these means!

If we can estimate the variance of
these distributions, we know the
variance of these means!

Steps to estimate Value of Information for prototype

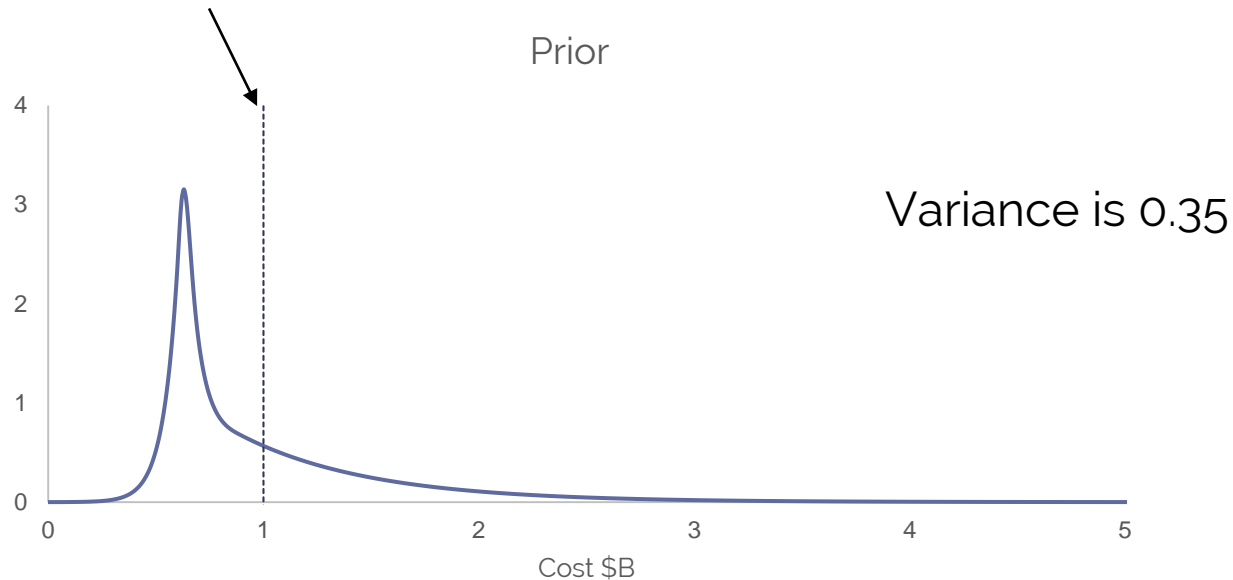
1. Estimate distribution of costs (prior)
2. Estimate reduction in variance of cost due to prototype (posteriors)
3. Model preposterior distribution as prior distribution with mean held constant (from 1) and variance equal to reduction in variance (from 2)
4. Perform decision tree on preposterior distribution with some given alternative cost

Example

From *Prototyping Defense Systems*

Derived prior distribution from percent cost overruns of systems without prototypes

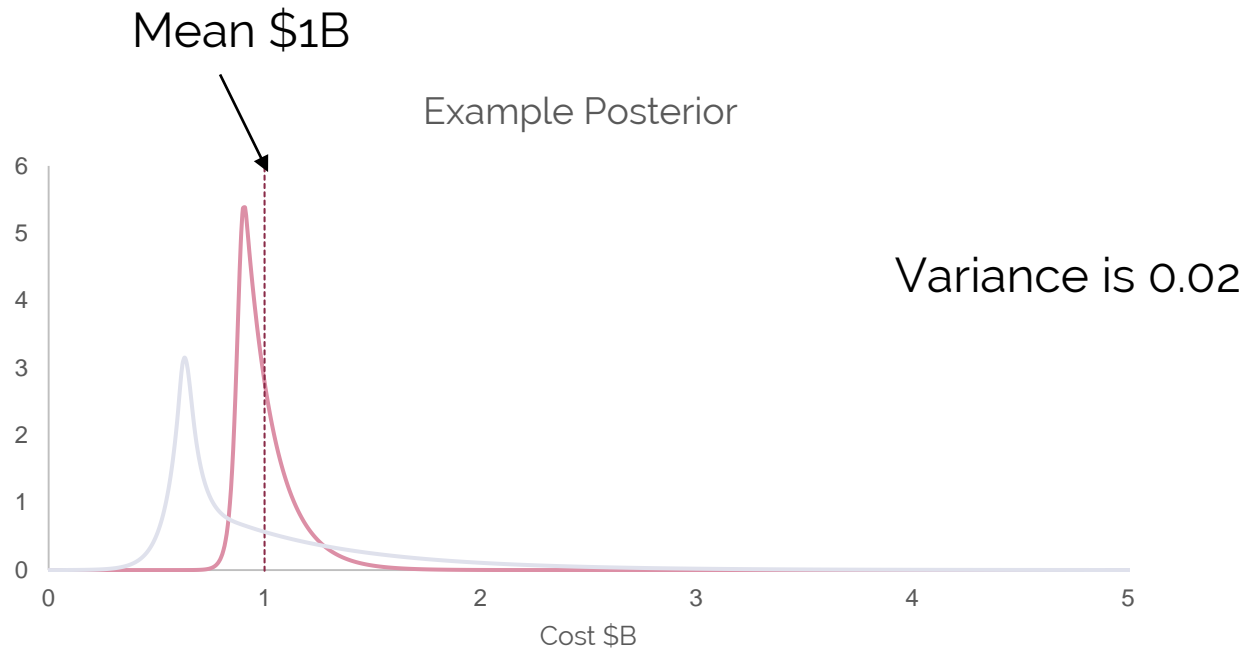
Normalized mean to \$1B



Example

From *Prototyping Defense Systems*

Derived a posterior variance from percent cost overruns of systems with prototypes



Example

From *Prototyping Defense Systems*

Derived a preposterior distribution as prior
with variance decreased by 0.02



Example

Suppose alternate cost is \$0.7B

For costs less than \$0.7B, find
partial expected value

$$\int_0^{0.7} xp(x)dx$$

Sum is cost
after
prototype
(\$0.665B)

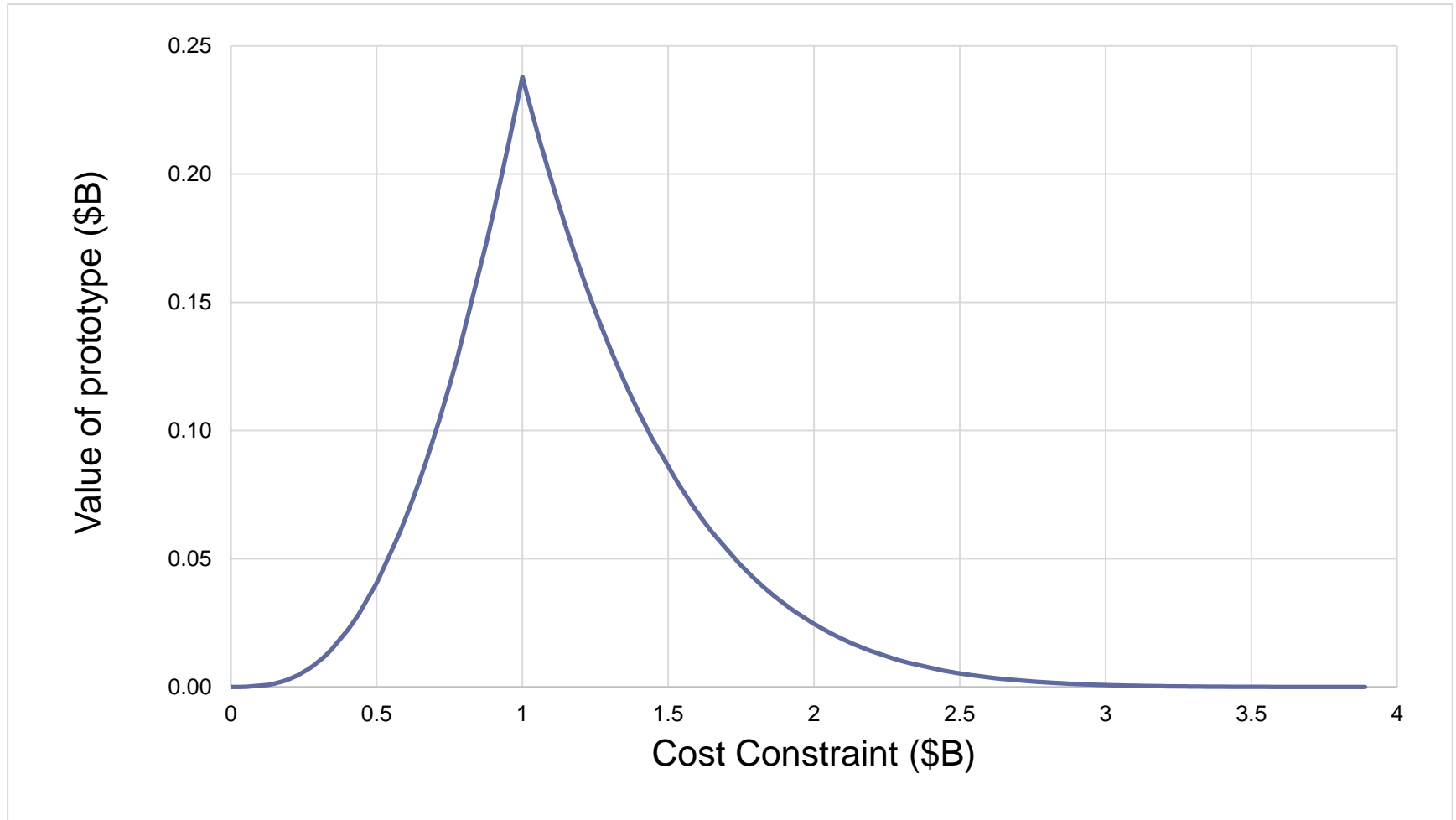
For costs more than \$0.7B,
find $0.7 * P(x > 0.7)$

Prototype
is worth
\$0.035B



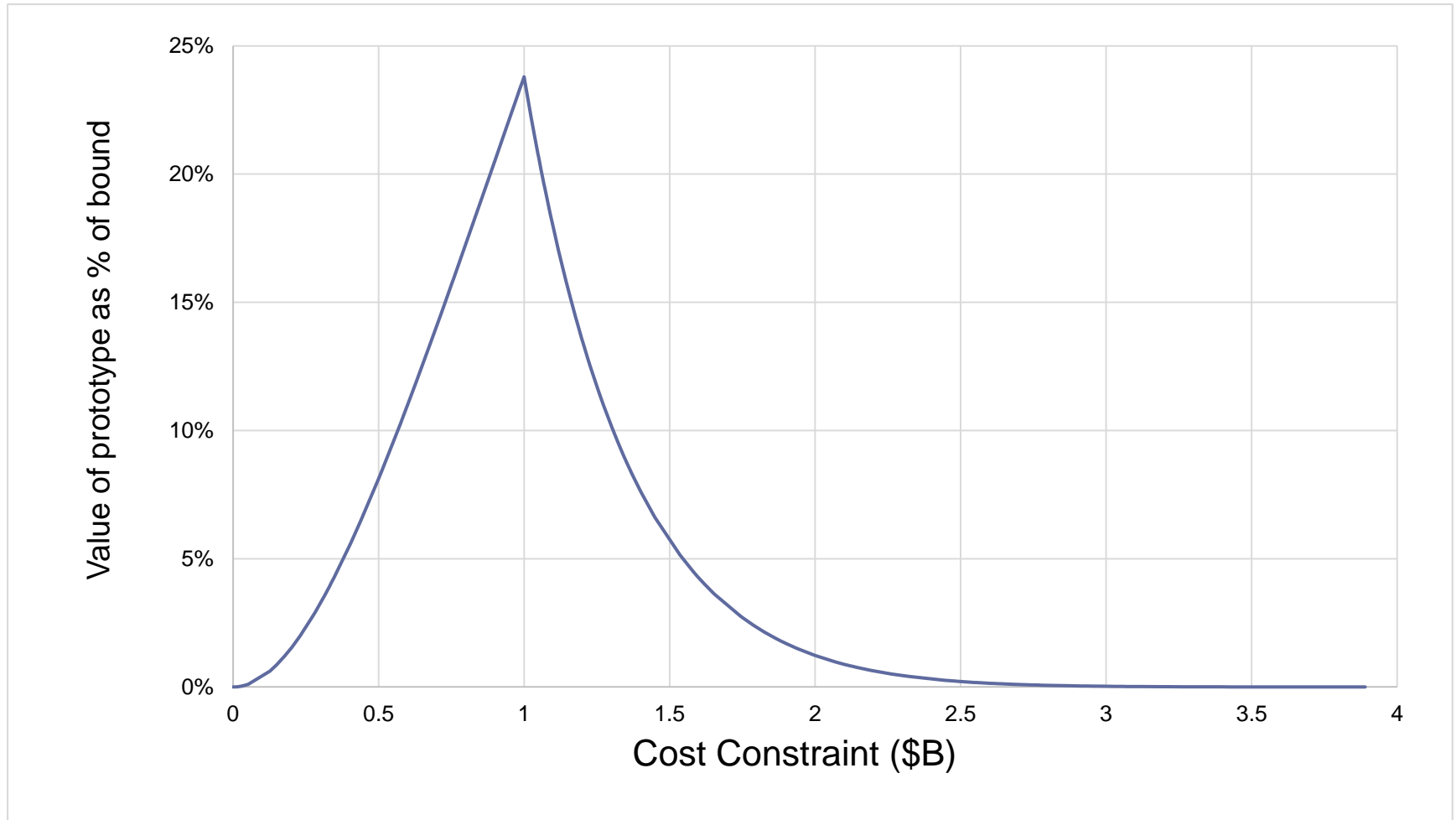
Example

Letting cost of alternative vary



Example

Letting cost of alternative vary



Thanks to the Air Force Research Laboratory for their
inspiration and help in this project.
And thanks to you!

Backup

Bayes tells us something about what we can expect

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

We want the distribution of

- the cost *given* some prototype outcome

We need the distribution of

- the prototype *given* cost
- the cost
- the prototypes

??????

Just a scaler!

So, not great.

But what if we look at attributes that incorporate all possible prototype outcomes?